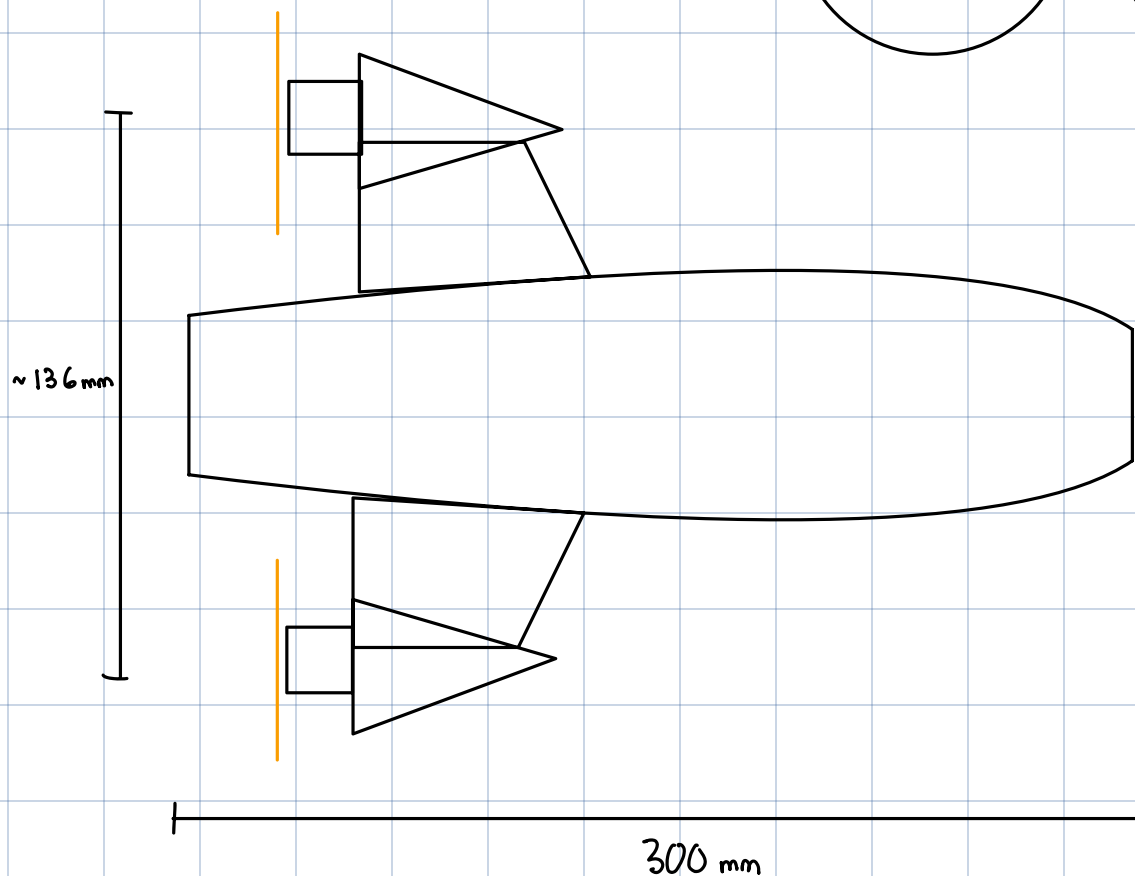
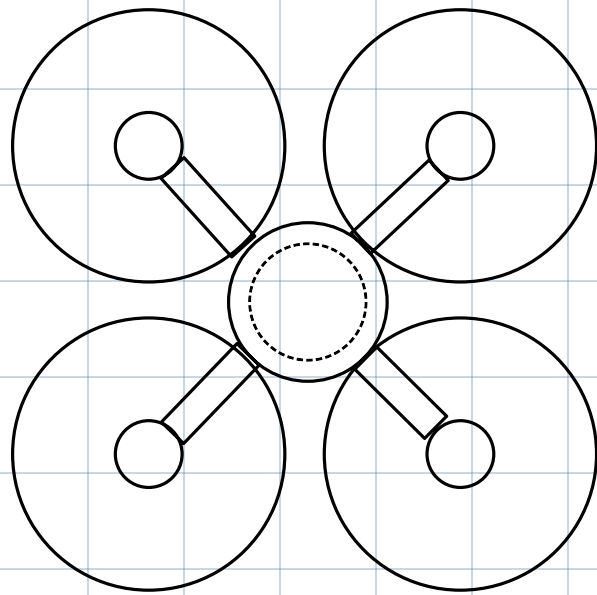


Johannes Kidder. High Speed Quad Drone

Hand Calculations *October 2026*



Based of estimations and product
descriptions and data. Not from
collected test data.



O. Variables & Constants (w/ est.'s)

Symbol	Value	What is it?	
D	129.5 mm	Prop diameter (T5146)	PS
p	0.1168 m	Prop Pitch	PS
a	68 mm	Motor offset	SW
d prop	44mm	Body diameter at prop plane	SW
k V	2050 rpm/v	Motor velocity constant	MS
V bat	22.2V nom 25.2V full	6s battery Pack	BS
ρ, μ	1.2 kg/m ³ , 1.81 x 10 ⁻⁵ Pa.s	Air at 20°C	S
C _T	0.12	Static thrust coefficient	Est.
C _{DA}	0.006 m ²	Drag area of quad	Est.
E _P	2410 MPa	PETG-CF modulus, printed	BDS
E _t	110,000 MPa	Carbon tube modulus	Est
ρ_P	1.25 g/cm ³ 60% infill	Printed PETG-CF	DS
L	69 mm	Fin length, body surfs to Motor axis	CAD
c _t	50 mm 10 mm	Fin chord, thickness	SW
Tubes	6mm OD, 4mm ID	Two spars at 13 and 29mm leading edge	SW
m tip	44g	Motor 36.8 g + Prop + motor cell + nut	M. S test

PS = Prop Spec., SW = Solid Works, MS = Motor spec.
BS = Battery spec., S = Standard, BDS = Bambu data sheet

1 Prop Clearances

$$R = D/2 = 129.5/2 = 64.8 \text{ mm}$$

$$r_m = \sqrt{a^2 + a^2} = \sqrt{68^2 + 68^2} = 96.2 \text{ mm}$$

$$\text{gap}_{pp} = 2a - D = 136 - 129.5 = 6.5 \text{ mm}$$

$$\text{gap}_{pb} = r_m - R - d_{prop}/2 = 96.2 - 64.8 - 22 = 9.4 \text{ mm}$$

2. Mass & Center of Gravity

Part	Mass m (g)	Station s (mm)	m.s (g·mm)
Motors, 4 x 36.8g	147.2	268	39,450
Props, 4 x 4.4g/est	17.6	282	4,963
Battery, CNHL 1300 6S	215.0	120	25,800
XT60 lead + strap (est.)	15.0	165	2,475
FC + ESC + capacitor (est.)	30.0	222	6,660
04 Air Unit (est.)	14.0	30	420
BeeID + receiver	6.0	60	360
Fuselage shell (est.)	88.0	140	12,320
Keel + tail ring (est.)	25.0	180	4,500
4 pods: fin, nacelle, tubes	113.0	235	26,555
Total	670.8		123,503

$$x_{cg} = \frac{\sum ms}{\sum m} = \frac{123,503}{670.8} = 184 \text{ mm}$$

$$W = mg = 0.6708 \times 9.81 = 6.58 \text{ N}$$

Notes: Center of Gravity sits 21 mm ahead of fins leading edge.

3. Hover thrust, RPM, power and flight time

Given: $W = 6.58 \text{ N}$, $D = 0.1295 \text{ m}$, $CT = 0.12$

$\rho = 1.2 \text{ kg/m}^3$, $kV = 2050 \text{ rpm/V}$, 1300 mAh at 22.2 V

Hover per Motor

$$T_h = \frac{W}{4} = \frac{6.58}{4} = 1.645 \text{ N}$$

$$D^4 = 0.1295^4 = 2.816 \times 10^{-4} \text{ m}^4$$

$$n_h = \sqrt{\frac{T_h}{CT \rho D^4}} = \sqrt{\frac{1.645}{0.12 \times 1.2 \times 2.816 \times 10^{-4}}} = 201 \text{ rev/s} = \underline{12100 \text{ rpm}}$$

Full throttle, Per motor

$$\text{RPM}_{\max} = 0.8 \times kV \times V = 0.8 \times 2050 \times 21 = 34,440 \text{ rpm}$$

$$T_{\max} = 0.12 \times 1.2 \times 574^2 \times 2.816 \times 10^{-4} = \underline{13.4 \text{ N}}$$

$$T/W = 4 \times 13.4 / 6.58 = 8.1$$

Hover Power (actuator-disk momentum theory) and flight time

$$A = \pi R^2 = \pi (0.0648^2) = 0.01318 \text{ m}^2$$

$$P_{\text{ideal}} = \frac{T_h^{3/2}}{\sqrt{2\rho A}} = \frac{1.645^{1.5}}{\sqrt{2 \times 1.2 \times 0.01318}} = 11.9 \text{ W}$$

$$P_{\text{elec}} = \frac{4 P_{\text{ideal}}}{FM \cdot \eta} = \frac{4 \times 11.9}{0.5 \times 0.8} = 119 \text{ W}$$

$$I = \frac{P}{V} = \frac{119}{22.2} = 5.3 \text{ A}$$

$$t = \frac{0.8 \times 1.3 \text{ Ah}}{5.3 \text{ A}} = 0.195 \text{ h} = 11.7 \text{ min}$$

Notes:

4. Top speed and Reynolds Number

Given: $T_o = 4 \times 13.4 = 53.4 \text{ N}$ (static thrust), $n_{\text{max}} = 574 \text{ rev/s}$

$\rho = 0.1168 \text{ m}$, $W = 6.58 \text{ N}$, $C_d A = 0.006 \text{ m}^2$ (est.), length = 0.3 m

$$V_p = n_{\text{max}} \rho = 574 \times 0.1168 = 67.1 \text{ m/s}$$

$$T(V) = T_o \left(1 - \frac{V}{V_p}\right), \quad D(V) = \frac{1}{2} \rho V^2 C_d A, \quad T(V), \quad T(V) = \sqrt{W^2 + D^2}$$

$V (\text{m/s})$	$T(V)$ available (N)	D (N)	$\sqrt{W^2 + D^2}$ needed (N)	Result
50	13.6	4.0	11.1	Still accelerating
52	12.0	4.7	11.7	Still accelerating
54	10.4	10.5	12.4	Too fast

Interpolate between 52 and 54 m/s

$$V_{top} \approx 52.2 \text{ m/s} = \frac{188 \text{ km/h}}{3.6}, \quad D(V_{top}) = 9.8 \text{ N}$$

$$Re = \frac{\rho V L}{\mu} = \frac{1.2 \times 52.2 \times 0.30}{1.81 \times 10^{-5}} = 1.0 \times 10^6$$

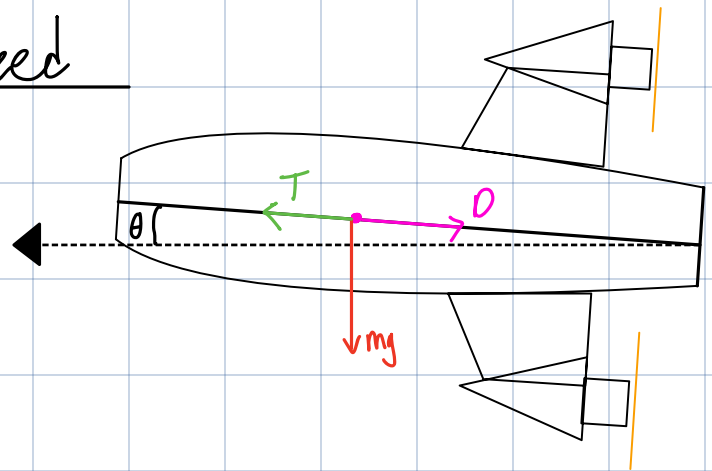
Notes: $Re = 10^6$ means the boundary layer is likely turbulent over its aft half. Smooth surface is vital, more than shape changes. $C_d A$ is the number to change. CFD gives one value. GPS top speed runs with this equation solved backward give a measured one. 2 blade speed props are worth testing to reduce $C_d A$.

5. Body angle at top speed

Given: $W = 6.58 \text{ N}$, $D = 9.8 \text{ N}$

$$\tan \theta = \frac{W}{D} = \frac{6.58}{9.8} = 0.671$$

$$\theta = 34^\circ \text{ nose } \uparrow, \quad T = \sqrt{W^2 + D^2} = 11.8 \text{ N}$$



Notes: Check if 34° is a hard boundary, the body will make lift at that angle, which carries W and lowers θ . Angle Camera down 20° to match the horizon and account for lift.

6. Fin sideways Natural frequency - first natural frequency f , for sideways bending

Given: fin section [round 5 mm nose, full 10 mm thickness from 5 mm to 34 mm, straight taper to 1 mm at 50 mm], holes $\varnothing 6.2$ at 13 and 24 mm and $\varnothing 5$ at 21 mm, tubes 6/4 mm in the two $\varnothing 6.2$ mm holes, $L = 69$ mm

$$E_p = 2910 \text{ MPa}, E_t = 110,000 \text{ MPa}, m_{tip} = 49 \text{ g}$$

Step 1: Area moment of inertia about the chord line

$$I_{flat} = \frac{wt^3}{12} = \frac{29 \times 10^3}{12} = 2,417 \text{ mm}^4$$

$$I_{nose} = \frac{\pi r^4}{8} = \frac{\pi 5^4}{8} = 245 \text{ mm}^4$$

$$I_{taper} = \int_0^{16} \frac{h(x)^3}{12} dx, h = 10 - \frac{9x}{16} \Rightarrow \frac{1}{12} \cdot \frac{16}{9} \cdot \frac{10^4 - 14^4}{4} = 370 \text{ mm}^4$$

$$I_{holes} = 2 \cdot \frac{\pi (6.2)^4}{64} + \frac{\pi (5)^4}{64} = 145 + 31 = 176 \text{ mm}^4$$

$$I_p = 2,417 + 245 + 370 - 176 = 2857 \text{ mm}^4$$

$$I_t = \frac{\pi (6^4 - 4^4)}{64} = 51.1 \text{ mm}^4 \text{ each}$$

Step 2: bending stiffness and tip spring rate

$$EI = E_p I_p + 2 E_t I_t = 2910(2857) + 2(110000)(51.1) = 19.5 \times 10^6 \text{ Nmm}^2$$

$$k = \frac{3EI}{L^3} = \frac{3(19.5 \times 10^6)}{69^3} = 178 \text{ N/mm} = 1.78 \times 10^5 \text{ N/m}$$

Step 3: effective mass & frequency

$$A_p = \underbrace{39.3}_{\text{nose}} + \underbrace{290}_{\text{flat}} + \underbrace{88}_{\text{taper}} - \underbrace{80}_{\text{holes}} = 337 \text{ mm}^2$$

$$m_{fin} = \underbrace{337(\text{mm}^2)(1.25 \times 10^{-3})}_{17.5 \text{ g okstr}}(0.6) + \underbrace{2(15.7)(69)(1.6 \times 10^{-3})}_{3.5 \text{ g tubes}} = 21.0 \text{ g}$$

$$m_{eff} = m_{tip} + \frac{33}{140} m_{fin} = 49 + 0.236(21.0) = 53.9 \text{ g}$$

$$f_1 = \frac{1}{2\pi} \sqrt{\frac{k}{m_{eff}}} = \frac{1}{2\pi} \sqrt{\frac{1.78 \times 10^5}{0.0539}} = 290 \text{ Hz}$$

Note: The Carbon tubes carry $\approx 58\%$ of the sideways stiffness.

7. Fin for-aft natural frequency - f_2 for bending fore-aft, along the chord

Given: fin, CF tubes, L and $m_{eff} = 53.9 \text{ g}$

Step 1: Neutral Axis

Piece	Area $A(\text{mm}^2)$	Centroid $x(\text{mm})$	Own $I(\text{mm}^4)$
Nose half circle, $r=5$	39.3	$5 - 4r/3\pi = 2.88$	$(\frac{\pi}{8} - \frac{8}{4\pi})r^4 = 69$
flat, 29×10	290	19.5	$t \frac{W^3}{12} = 20,324$
Taper, 16 long, $10 \rightarrow 1$	88.0	39.82	$h^3(a^3 + 4ab + b^3)/36(a+b) = 1,458$
Hole $\varnothing 6.2$ (subtract)	30.2	13.00	72
Hole $\varnothing 5$ (subtract)	19.6	21.00	31
Hole $\varnothing 6.2$ (subtract)	30.2	29.00	72
Tubes, 2×15.7 (Carbon)	31.4	13 & 29	2×51.1

$$A_p = 39.3 + 290 + 88 - 80.0 = 337.3 \text{ mm}^2, \quad Q_p = \sum A x = 7,592 \text{ mm}^3$$

$$\bar{x} = \frac{E_p Q_p + E_f A_f (13+29)}{E_p A_p + 2 E_f A_f} = \frac{2910(7592) + 110,000(15.7)(42)}{110,000(31.4)} = 21.3 \text{ mm}$$

Step 2: moment of inertia about $\bar{x} = 21.3 \text{ mm}$ ($I = I_{own} + A d^2$)

$$I_p = 13,445 + 21,297 + 37,537 - 4,049 = 62,230 \text{ mm}^4$$

$$I_t = 2(51.1) + 15.7(8.33^2 + 7.67^2) = 2,116 \text{ mm}^4$$

Step 3: stiffness & frequency, $k = \frac{3EI}{L^3} = \frac{3(414 \times 10^6 \text{ Nmm}^2)}{64^3} = 3780 \text{ N/mm}$

$$EI = 2910(62,230) + 110,000(2,116) = 414 \times 10^6 \text{ Nmm}^2$$

$$f_2 = \frac{1}{2\pi} \sqrt{\frac{3.78 \times 10^2}{0.0534}} = 1,330 \text{ Hz}$$

Notes: Plastic carries 44%, Spreading tubes apart raises f_2 fastest, because l_t grows with the square of their distance from $\bar{x}$.

8. Resonance Check (Campbell diagram) - Speed where (force $f = f_n$)

Given: fin modes $f_1 = 290 \text{ Hz}$ (sideways), $f_2 = 1330 \text{ Hz}$ (fore-aft)
motor speed from 12,100 rpm (hover) to 34,440 rpm (full throttle),

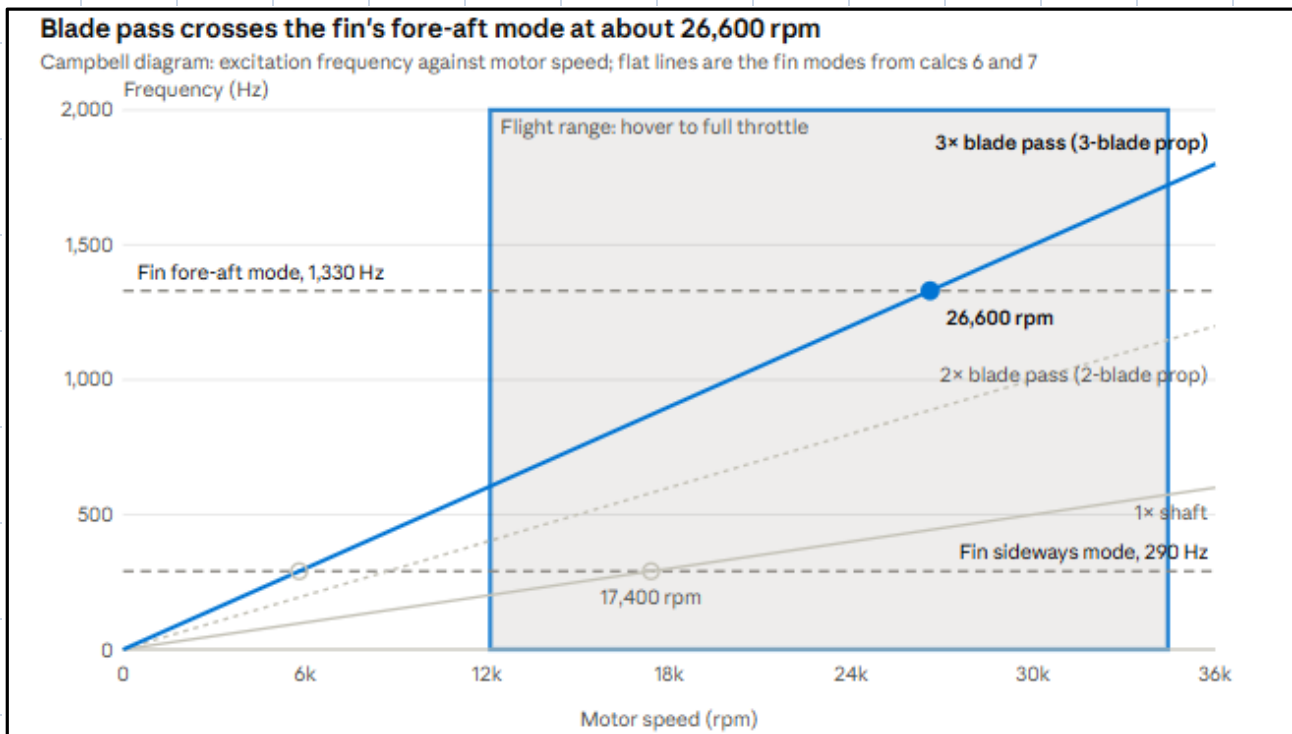
$$f_{exc} = N \cdot \frac{\text{rpm}}{60} \Rightarrow \text{rpm}_{cross} = \frac{60 f_n}{N}$$

$$\text{Fore-aft, blade pass: } \frac{60(1330)}{3} = 26,600 \text{ rpm} = 77\% \text{ throttle}$$

$$\text{Sideways, shaft: } \frac{60(290)}{1} = 17,400 \text{ rpm (mid throttle)}$$

$$\text{Sideways, blade pass: } \frac{60(290)}{3} = 5,800 \text{ rpm (below hover)}$$

$$\text{Full-throttle blade pass: } \frac{3(34,440)}{60} = 1,722 \text{ Hz} = 1.29 f_2$$



Notes: Full throttle is fine, because Blade pass sits 29% above f_2 . The risk is holding at 70-85% throttle where blade pass

sits on the force-aft mode. to push f_2 above, $1.2 \times 1721 = 2070 \text{ Hz}$
EI must grow by $\left(\frac{2070}{1330}\right)^2 = 2.4 \times$. 2 blade propeller never reaches
fore aft mode.

9. Fin root Stress - Bending stress from thrust, sideways tip load to reach 35MPa

Given: full throttle thrust $T = 13.4 \text{ N}$ per motor (calc 3), $L = 69 \text{ mm}$

$EI = 414 \times 10^6 \text{ N}\cdot\text{mm}^2$ fore aft and $19.5 \times 10^6 \text{ N}\cdot\text{mm}^2$ sideways, Neutral axis at 21.3 mm

Printed PETG-LF strength 35 MPa.

a) Thrust load, bending fore-aft

$$M = TL = 13.4 \times 69 = 922 \text{ Nmm}$$

$$\sigma_{\text{photo}} = \frac{M_y E_p}{EI} = \frac{922(50 - 21.3)(2910)}{414 \times 10^6} = \underline{0.19 \text{ MPa}}$$

$$\sigma_{\text{tube}} = \frac{M_y E_t}{EI} = \frac{922(8.33 + 3)(110,000)}{414 \times 10^6} = \underline{2.8 \text{ MPa}}$$

b) Sideways load to reach 35 MPa (outer fiber at $y = \frac{t}{2} = 5 \text{ mm}$)

$$M_{\text{allow}} = \frac{\sigma EI}{y E_p} = \frac{35(19.5 \times 10^6)}{5(2910)} = 47,000 \text{ Nmm}$$

$$f_{\text{allow}} = \frac{M_{\text{allow}}}{L} = \frac{47000}{69} = \underline{680 \text{ N}}$$

Notes: Thrust stress are under 1% of what material can withstand. So the fin is sized by stiffness (calc 6-8) not strength. Weakness is printed fin joint. & heat inserts.

10. Motor Cant side force - Sideways force each canted motor makes

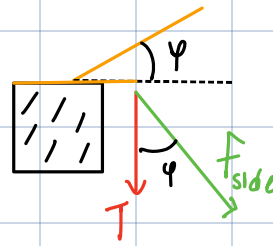
Given: $T = 13.4 \text{ N}$ per motor at full throttle; cant angles $\varphi = 5^\circ, 6^\circ, 8^\circ$

$$F_{\text{side}} = T \sin \varphi, \quad \text{axial loss} = 1 - \cos \varphi$$

$$\varphi = 5^\circ: F_{\text{side}} = 1.16 \text{ N}, \quad \text{Loss} = 0.4\%$$

$$\varphi = 6^\circ: F_{\text{side}} = 1.4 \text{ N}, \quad \text{Loss} = 0.5\%$$

$$\varphi = 8^\circ: F_{\text{side}} = 1.86 \text{ N}, \quad \text{Loss} = 1.0\%$$



Notes: at 6° each motor gains 1.4N side force, about 10% of its thrust for a half of a percent of thrust lost. That side force at 96mm from the axis gives the flight controller a roll moment about the flight path, the axis a plain quad controls weakly, at almost no cost in top speed. A thrust stand cannot measure this; compare roll response in logs with and without cant.

11. Battery fit. - the minimum body diameter over the pack's length.

Given: CNHL 1300 Gs pack, listed $82 \times 40 \times 36 \text{ mm}$, centered on the fuselage at station 120 (spans 79-161mm); shell wall 1.5mm; 24mm at nose, 65 at 105mm, 56mm at 222mm.

$$d_{\text{pack}} = \sqrt{40^2 + 36^2} = 53.8 \text{ mm}$$

$$D_{\text{inner}} \geq 53.8 + 2(1) = 55.8 \text{ mm}, \quad D_{\text{outer}} \geq 55.8 + 2(1.5) = \underline{58.8 \text{ mm}}$$

$$D(161) \geq 65 - \frac{161-105}{222-105} (65-56) = 60.7 \text{ mm}$$

$$D(79) \geq 24 + \frac{79}{105} (65-24) = 54.8 \text{ mm} \quad (\text{straight line connection})$$

Note: Measure Battery pack with Calipers.

12. Thrust stand sensing - a voltage divider that keeps the ADC under 5V, current sensor's output at peak and its resolution. The power and efficiency formulas the logger will use.

Given: Gs pack, 2S.2V full, Arduino Uno R4, ACS78LCB-100B current sensor

20 mV/A, 2.5V output at 0A on a 5V supply; peak motor current 49.5A.

a) Voltage divider, $R_1 = 47\text{k}\Omega$ (top), $R_2 = 10\text{k}\Omega$ (bottom)

$$V_{\text{out}} = V_{\text{in}} \frac{R_2}{R_1 + R_2} = 25.2 \frac{10}{57} = 4.42\text{V} < 5\text{V}$$

$$I = \frac{25.2}{57\text{k}\Omega} = 0.44\text{mA}, \quad P_{R_1} = I^2 R_1 = 9\text{mW} \ll 0.25\text{W}$$

$$\text{ADC step} = \frac{5}{2^{14}} = 0.305\text{mV} \Rightarrow \text{Pack step} = 0.305 \times 5.7 = 1.7\text{mV}$$

b) Current sensor

$$V_{\text{out}}(49.5\text{V}) = 2.5 + 0.020(49.5) = 3.49\text{V} < 5\text{V}$$

$$\text{Current step} = \frac{0.305\text{mV}}{20\text{mV/A}} = 0.015\text{A per count}$$

c) Logged quantities:

$$P = VI(W), \quad \eta_{\text{thrust}} = \frac{T(g)}{P(W)} \left(\frac{\text{g}}{\text{W}} \right), \quad V = \text{counts} \times \frac{5}{2^{14}} \times \frac{R_1 + R_2}{R_2}$$

Notes: real resolution is set by noise, not by step size:

Measure R_1 and R_2 with a meter and use the measured ratio because 1% resistors can shift voltage reading by up to 1.6%.

13. Thrust stand Calibration and uncertainty - Example data for arithmetic

Given: known mass hung on load cell,

Mass (g)	Force g (N)	Reading x (counts)
0	0.00	12
200	1.962	8420
400	3.924	16,880
600	5.886	25,250
800	7.848	33,700
1,000	9.810	42,090

- Find Calibration $y = b_0 + b_1 x$, its SE, and 95% uncertainty of one thrust reading

Step 1: least squares fit ($n=6$)

$$\bar{x} = 21,054, \quad \bar{y} = 4.905 \text{ N}$$

$$S_{xx} = \sum (x_i - \bar{x})^2 = 1.240 \times 10^4, \quad S_{xy} = \sum (x_i - \bar{x})(y_i - \bar{y}) = 2.840 \times 10^{-5}$$

$$b_1 = \frac{S_{xy}}{S_{xx}} = 2.331 \times 10^{-9} \text{ N/rev}, \quad b_0 = \bar{y} - b_1 \bar{x} = -0.0037 \text{ N}$$

Step 2: Standard error and 95% confidence interval

$$s = \sqrt{\frac{SSE}{n-2}} = \sqrt{\frac{9.51 \times 10^{-5}}{4}} = 0.0049 \text{ N}$$

$$\hat{y}_0 = b_0 + b_1(30,000) = 6.989 \text{ N}$$

$$PI = \hat{y}_0 \pm t_{0.025, 4} s \sqrt{1 + \frac{1}{n} + \frac{(x_0 - \bar{x})^2}{S_{xx}}} = 6.989 \pm 2.776(0.0049)(1.10) = \underline{6.99 \pm 0.015 \text{ N}}$$

Step 3: measured thrust coefficient for calc. 3

- With thrust T_i logged at rotation rates n_i (rev/s), fit $T = \beta n^2$ through origin

$$\beta = \frac{\sum T_i n_i^2}{\sum n_i^4}, \quad C_T = \frac{\beta}{\rho D^4}$$

Notes: $\pm 0.015 \text{ N}$ is the calibration scatter, spinning prop adds vibration and air flow noise, so uncertainty on running thrust will be larger.

Sources

- Allegro ACS758 data sheet: 100B variant, 20 mV/A, 2.5 V zero output at 5 V supply, $\pm 100 \text{ A}$. Allegro lists it as not for new design (ACS772/773 replace it).
- Bambu Lab PETG-CF technical data sheet: printed modulus and strength, as used in the design guide.

- **CNHL Black Series 1300 6S 2-pack: listed size and mass, checked Oct 11, 2026.**
- **T-Hobby Velox V2207 V3 (Mepsking): 2050KV, 36.8 g, checked Oct 11, 2026.**
- **Arduino UNO R4 WiFi ADC resolution: 14-bit analog reads; the 5 V default reference was not re-checked for this page.**
- **Geometry and design values: SolidWorks master model and the Teardrop Speed Quad Design Guide.**